

Exam geometry 2026

All exercises have equal weight so please try them all. You may refer to theorems and definitions of the lecture notes but are otherwise expected to prove any claims you make. Always explain your answers. Good luck!

1. Consider four independent vectors a, b, c, d in $\mathbb{R}^4$. We say a basis v_1, v_2, v_3, v_4 for $\mathbb{R}^4$ is special if $v_1 = a$ and $\{v_1, v_2, v_3, v_4\} = \{a, b, c, d\}$. Write down all six special bases explicitly and group those together that have the same orientation.
2. Consider $\mathbb{R}^2$ oriented such that e_2, e_1 is a positive basis and set $v = e_1 + e_2$ and $w = e_1 - e_2$. Find a Euclidean isometry of $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ such that $\angle(F(v), F(w)) = \pi$.
3. With respect to spherical distance in $\mathbb{S}^{100}$, which point is closer to $p = \frac{1}{10} \sum_{k=1}^{100} e_k$, the point $q = \frac{1}{2}(-e_0 + e_3 + e_6 + e_{99})$ or the point $r = \frac{1}{2}(e_1 + e_3 + e_6 - e_{100})$?
4. Explicitly describe three spherical lines A_1, A_2, A_3 in $\mathbb{S}^5$ such that for all $i \neq j$ we have $A_i \cap A_j = \emptyset$.
5. Recall that an affine subspace of a vector space V is a subset of the form $v + W$ where W is a linear subspace of V . Now assume V is finite dimensional of dimension at least 5. For any linear subspace $U \subseteq V^*$ define $U' = \{v \in V : f(v) = 1 \ \forall f \in U\}$. Prove or provide a counter example to the following statement: If $\dim U > 3$ then U' is an affine subspace of V .
6. Consider the linear subspace $W = \text{Span}(w_1, w_2, \dots, w_k)$ in vector space V and define $\omega = w_1 \wedge w_2 \wedge \dots \wedge w_k \in \Lambda^k V$. Prove that if $v \in W$ then $v \wedge \omega = 0$.
7. In $\Lambda \mathbb{R}^4$ simplify the following expression:

$$(1 + e_1 \wedge e_2) \wedge (1 + e_2 \wedge e_3) \wedge (1 + e_3 \wedge e_4) \wedge (1 + e_4 \wedge e_1)$$

8. Consider the spherical lines $A = \mathbb{S}^2 \cap a^\perp$, $B = \mathbb{S}^2 \cap b^\perp$ and $C = \mathbb{S}^2 \cap c^\perp$ in $\mathbb{S}^2$, where $a = e_2$, $b = 2e_1 + e_2$ and $c = e_1$. Using Clifford algebra, explicitly describe the spherical line obtained by first reflecting C into A and then reflecting the result in line B .
9. In terms of the Clifford algebra $\mathcal{C} = \mathcal{C}((\mathbb{R}^3)^*)$ for the Euclidean plane explain how two parallel Euclidean lines can be described by elements of $a, b \in \mathcal{C}$ and in what sense the product ab represents their intersection point "at infinity".